Technical University of Lodz
Institute of Electronics

Radius Estimation in Angiograms using Multiscale Vesselness Function

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iccs

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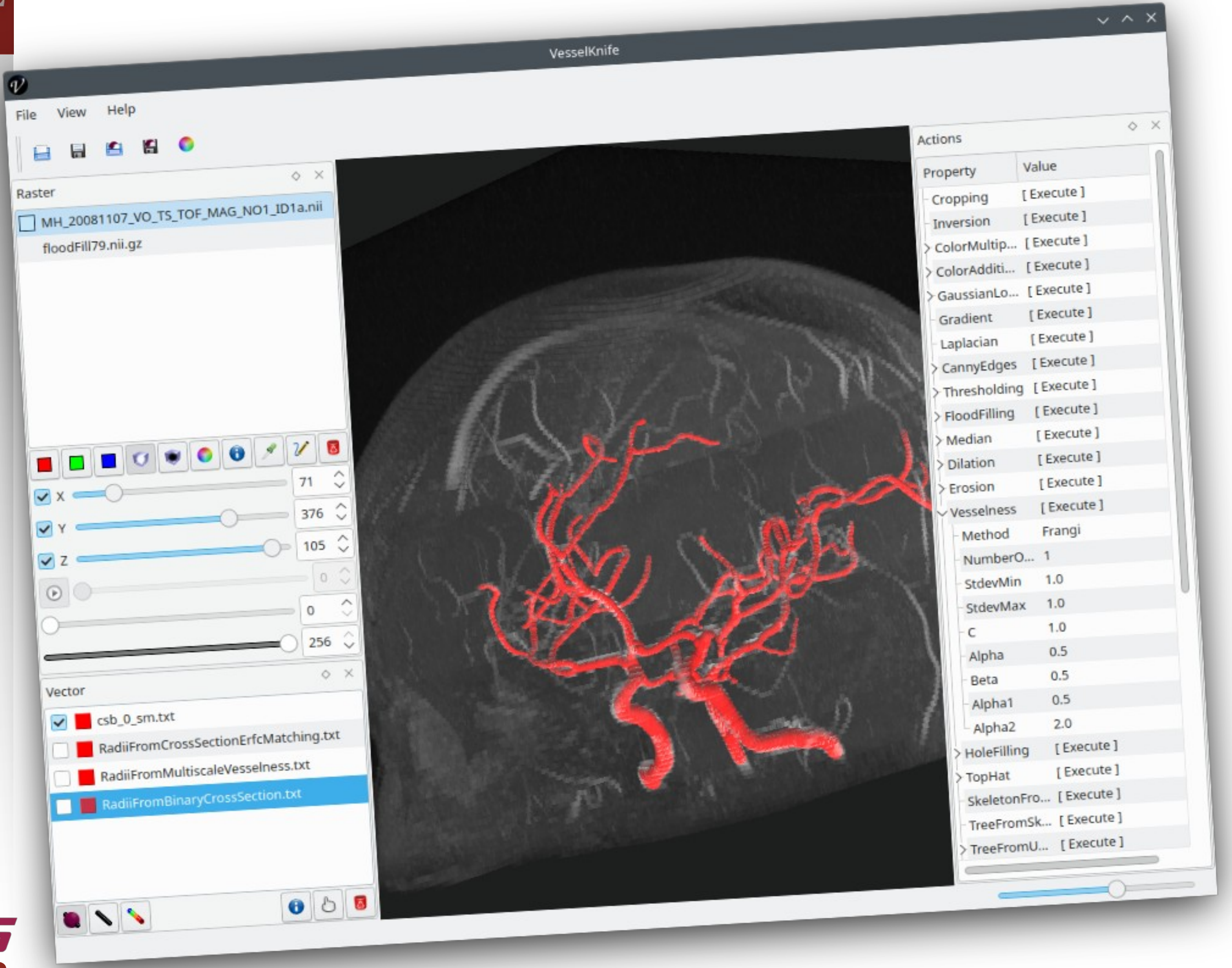
Previous work and motivation



2013-2016:
NCN 2013/08/M/ST7/00943
Numerical modeling of the cerebral venous and arterial system on a macro- and mesoscopic scale from three-dimensional magnetic resonance images

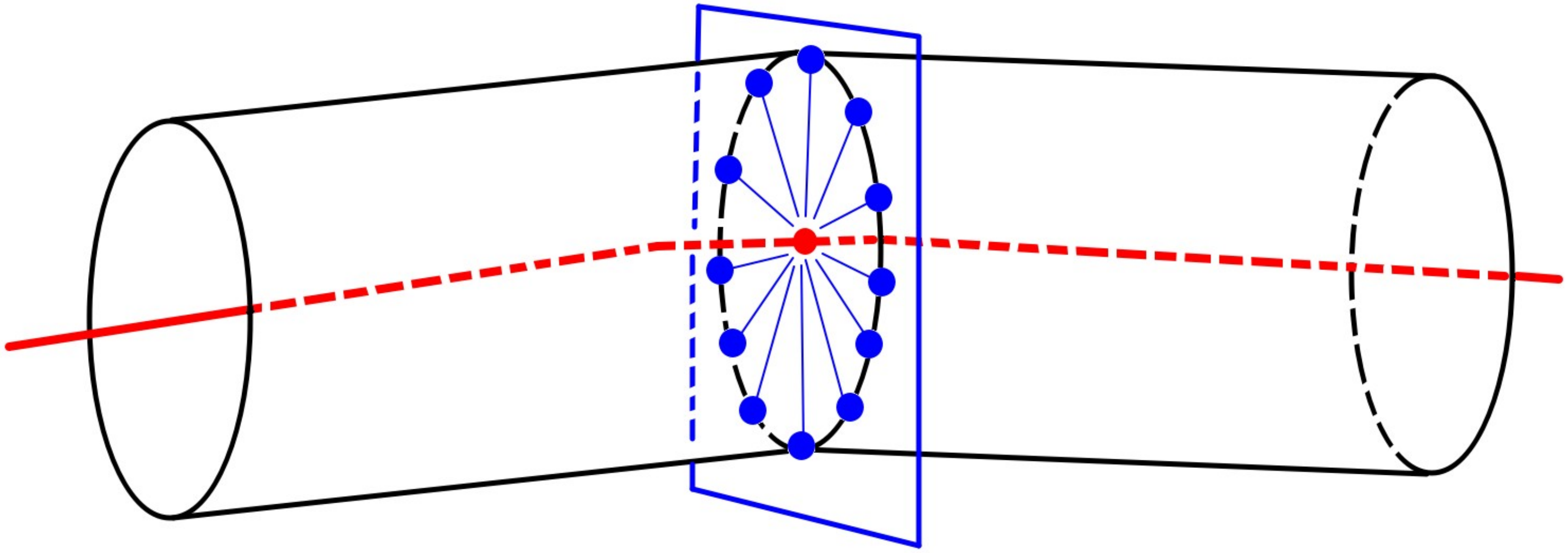
2015-2017:
NCN ST7/OPUS-8
The development of numerical methods for modeling and evaluation of renal perfusion using magnetic resonance imaging.

NCN – National Science Centre in Poland

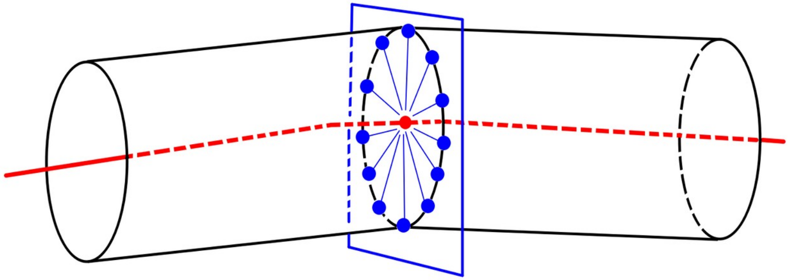
MRA Input

MRA Input

Ray-casting approach

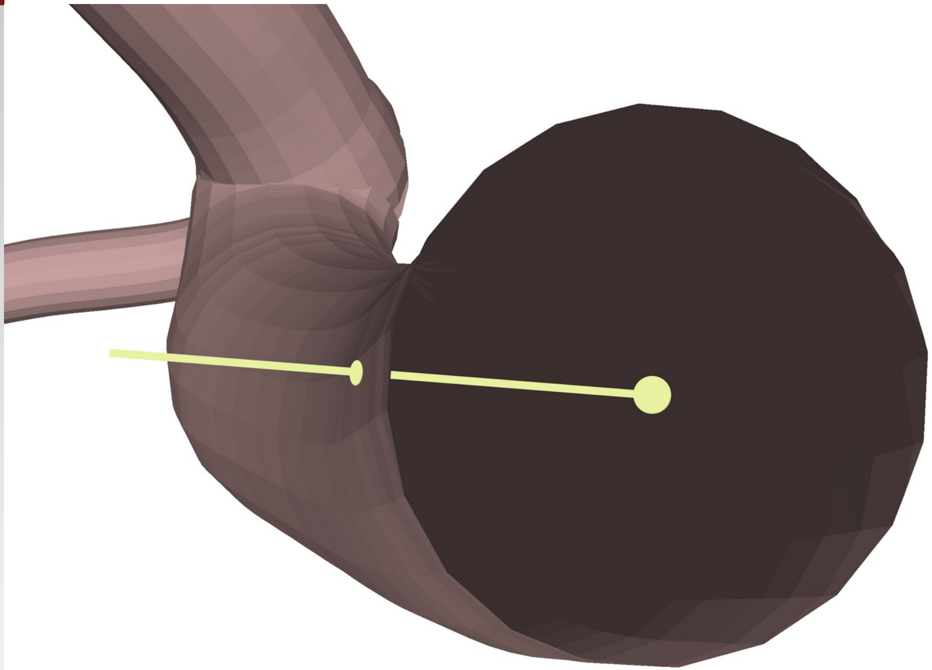


Ray-casting approach



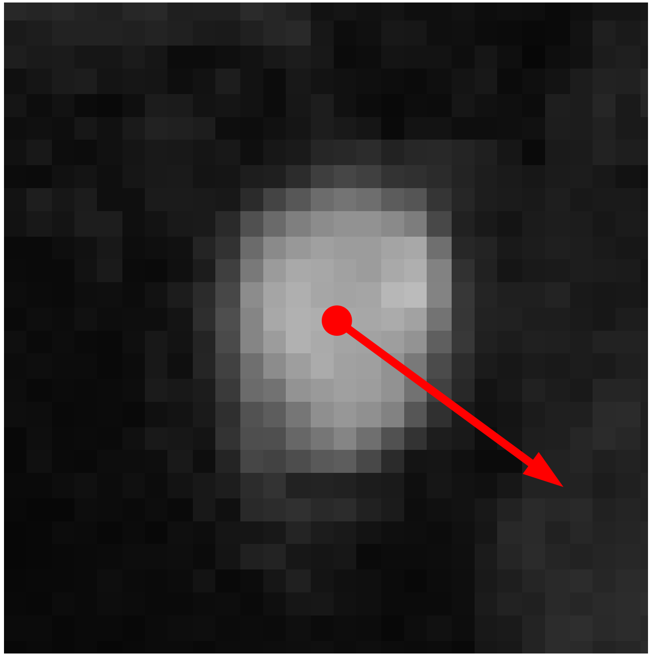
Requirements:
1. finding a centerline (center point + direction),
2. finding a cross-section orthogonal to the centerline,
3. the method of locating the vessel wall.

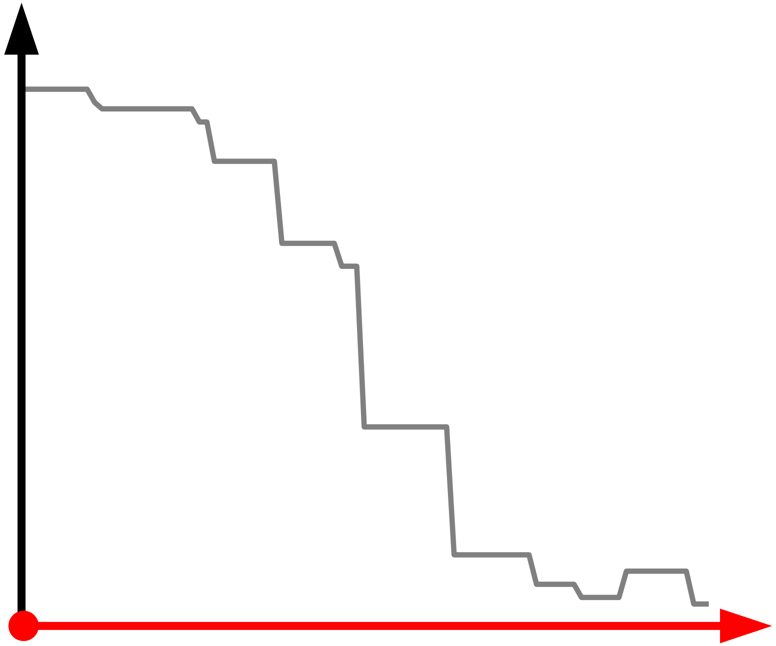
Locating the vessel wall



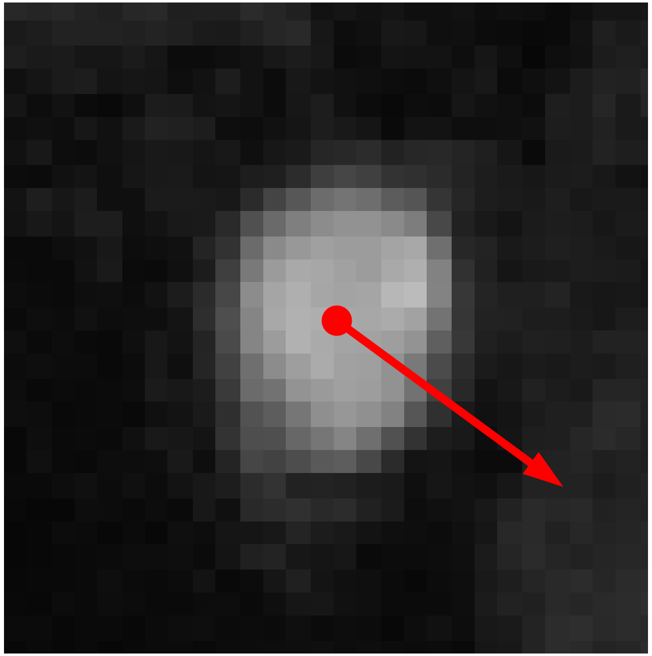
Requirements:
1. building a 3D surface model,
2. finding intersection of the ray with the wall.

Locating the vessel wall



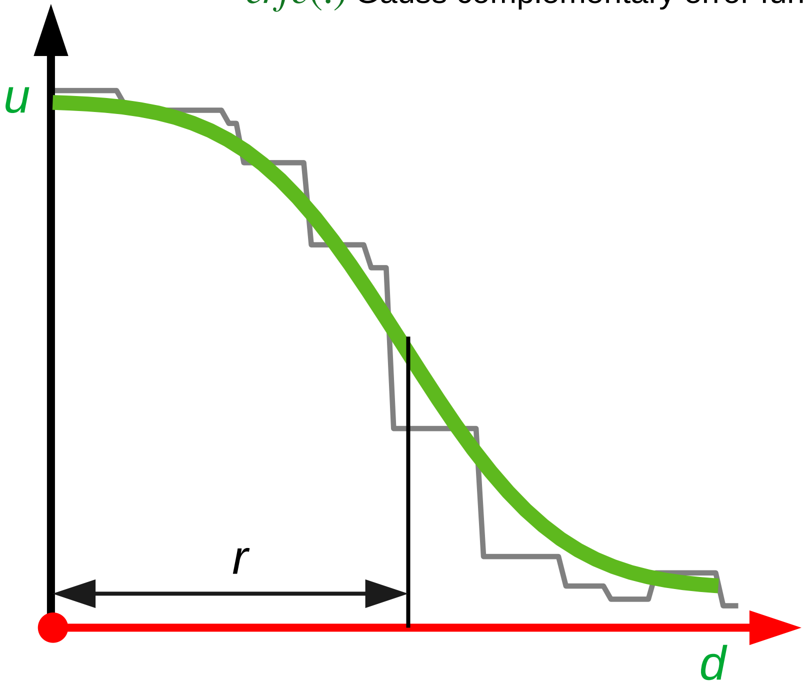


Locating the vessel wall



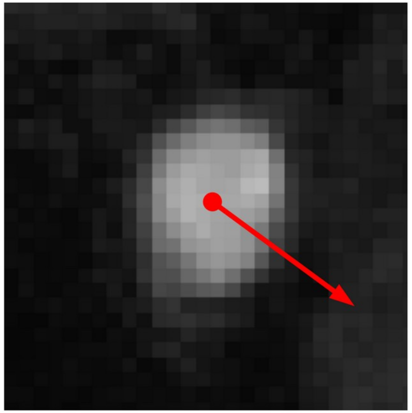
$$u(d;\Delta_V,\Delta_R,V_0,r) = V_0+\Delta_V\,erfc\left(\frac{(d-r)}{\Delta_R}\right)$$

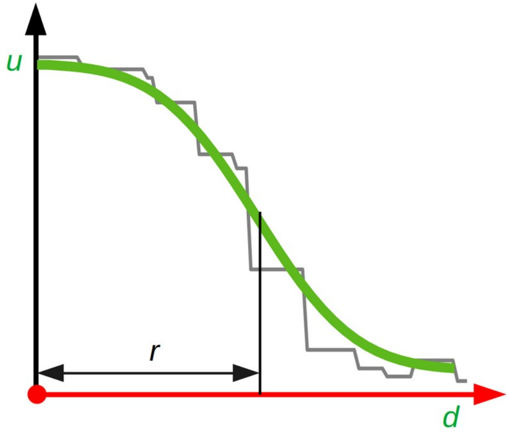
$erfc(.)$ Gauss complementary error function



Andrzej Materka, et al. *Automated modeling of tubular blood vessels in 3D MR angiography images*. 9th International Symposium on Image and Signal Processing and Analysis (ISPA). IEEE, 2015.

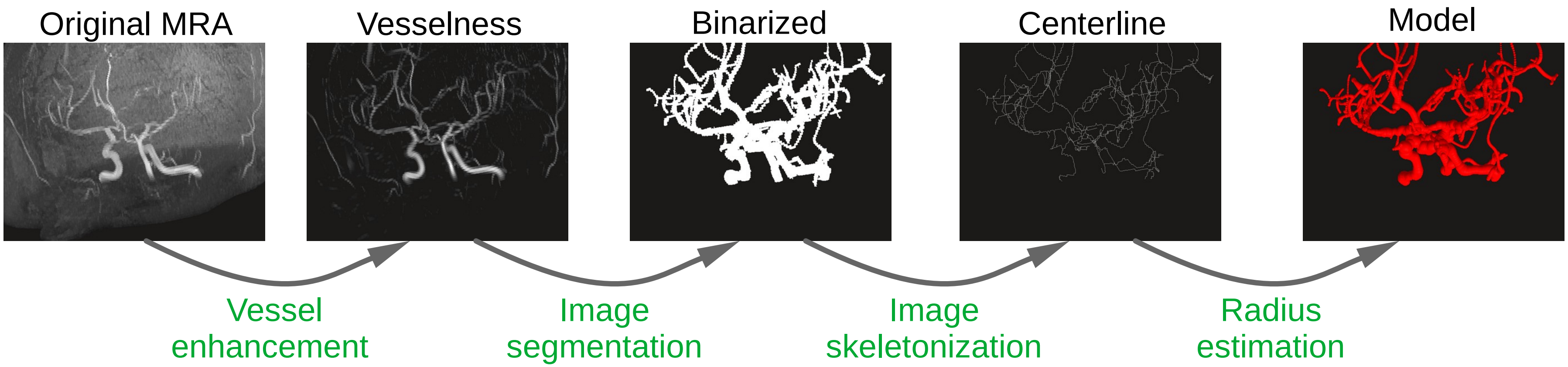
Locating the vessel wall



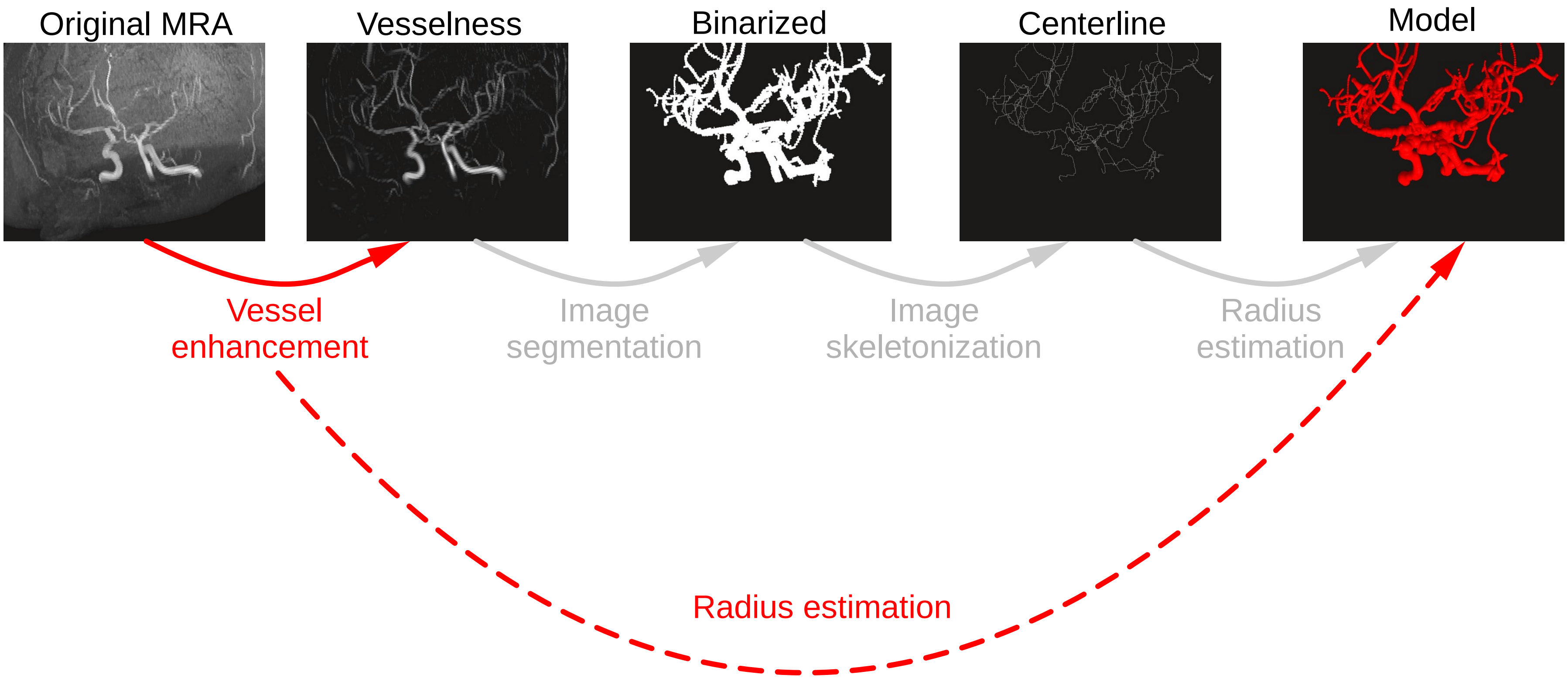


Requirements:
1. computation of cross-sectional image,
2. finding brightness profile,
3. fitting the *erfc* to the profile.

The goal


Original MRA
Vesselness
Binarized
Centerline
Model
Vessel enhancement
Image segmentation
Image skeletonization
Radius estimation

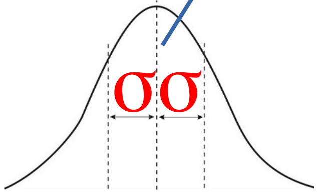
The goal


Original MRA
Vesselness
Binarized
Centerline
Model
Vessel enhancement
Image segmentation
Image skeletonization
Radius estimation
Radius estimation

Vesselness function

Smoothing

$$*$$



Hessian computation

$$(\mathbf{H}_f)_{i,j} = \frac{\partial^2 f}{\partial x_i\, \partial x_j}$$

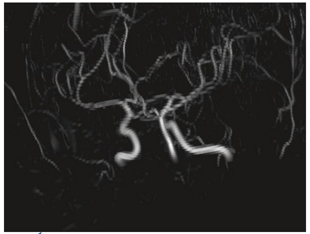
Eigen decomposition

$$\lambda_1^\star \geqslant \lambda_2^\star \geqslant \lambda_3^\star$$

$$F = \sigma^2 \begin{cases} \lambda_c^\star \exp\left(-\frac{{\lambda_1^\star}^2}{2(\alpha_1 \lambda_c^\star)^2}\right) & \lambda_1^\star \leqslant 0, \lambda_c^\star \neq 0 \\ \lambda_c^\star \exp\left(-\frac{{\lambda_1^\star}^2}{2(\alpha_2 \lambda_c^\star)^2}\right) & \lambda_1^\star > 0, \lambda_c^\star \neq 0 \\ 0 & \lambda_c^\star = 0 \end{cases}$$

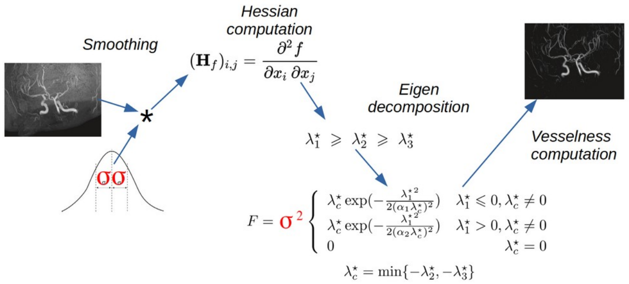
$$\lambda_c^\star = \min\{-\lambda_2^\star, -\lambda_3^\star\}$$

Vesselness computation

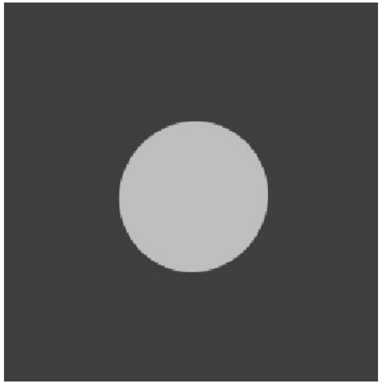


Sato, Yoshinobu, et al. *3D multi-scale line filter for segmentation and visualization of curvilinear structures in medical images.* CVRMed-MRCAS'97. Springer, Berlin, Heidelberg, 1997.

Vesselness function

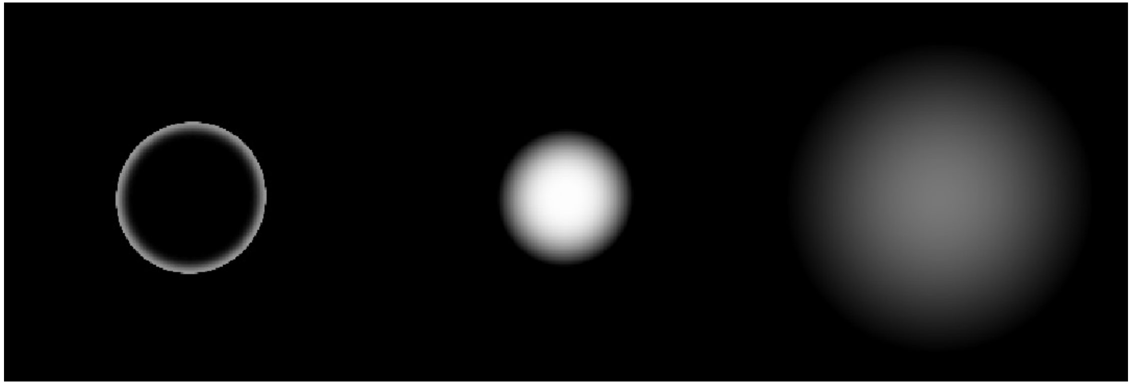


Cross section



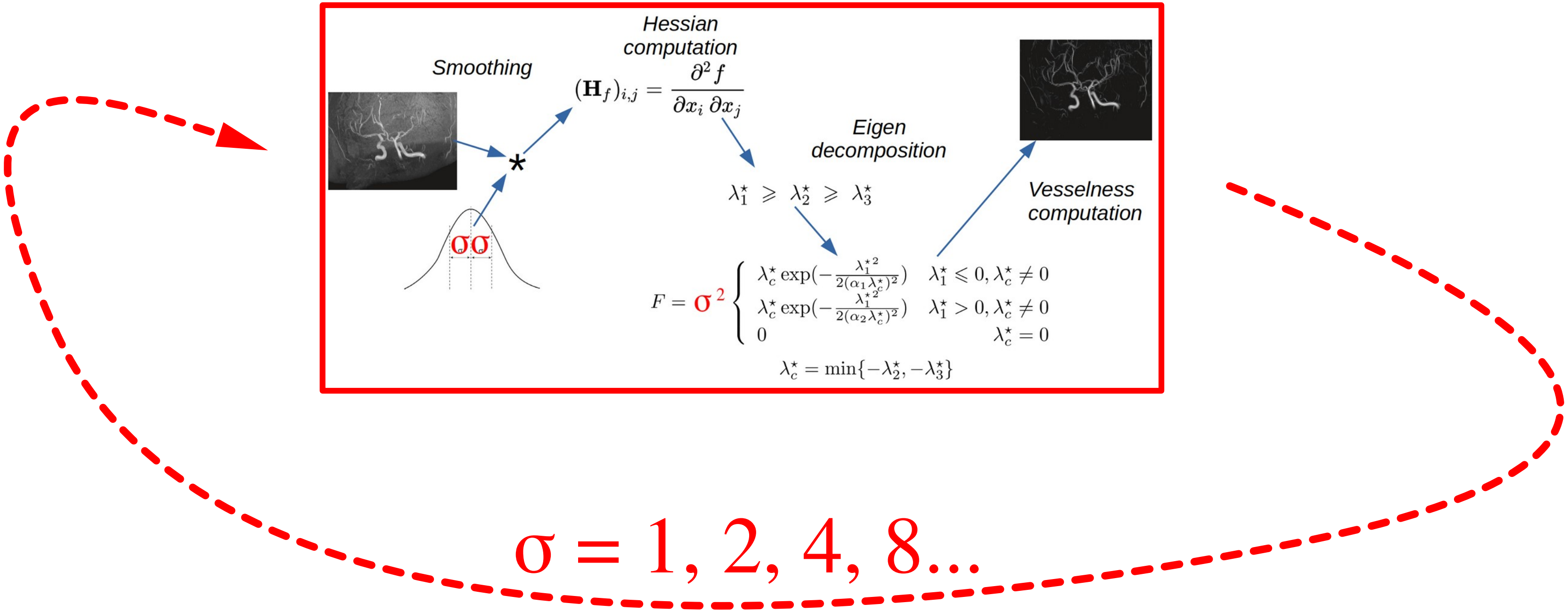
$r = 5$

F(σ)

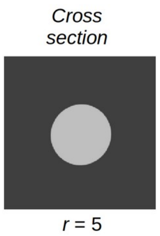


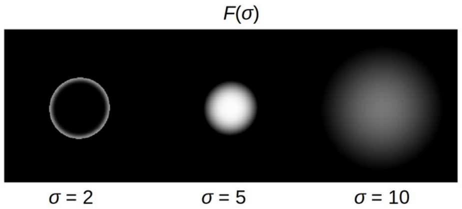
$\sigma = 2$ $\sigma = 5$ $\sigma = 10$

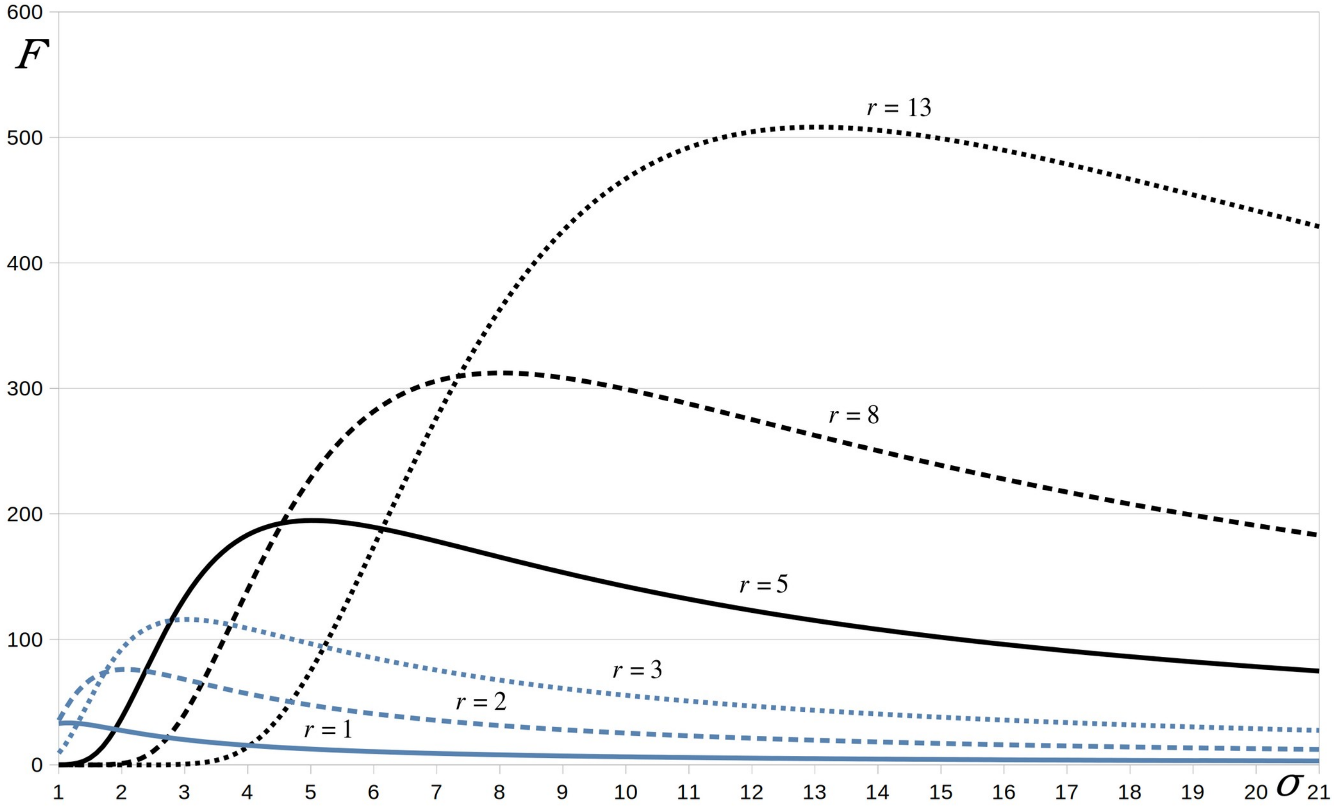
Multiscale vesselness



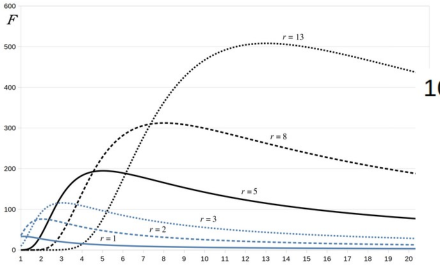
Sigma to radius relation

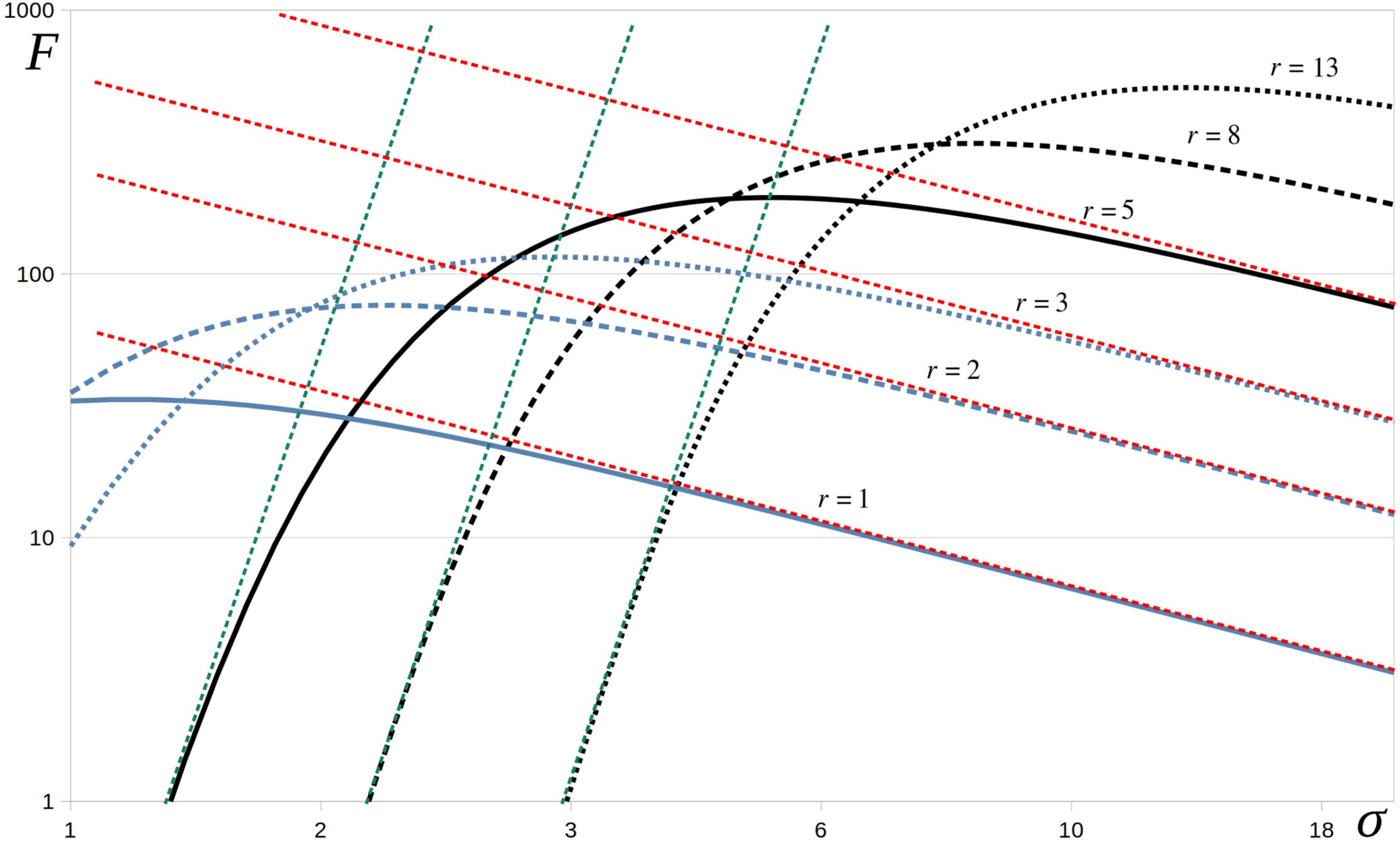




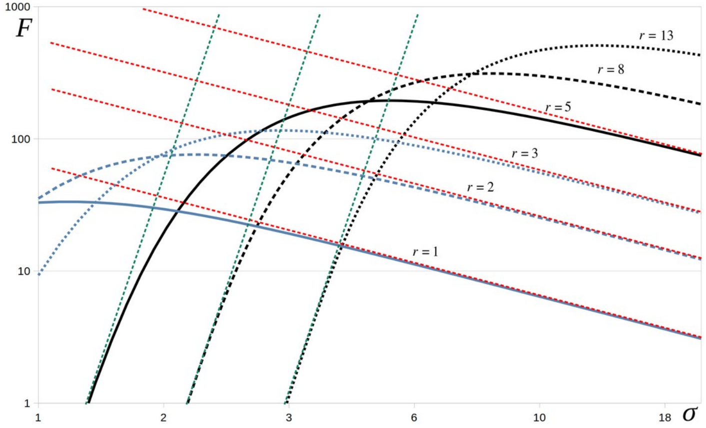


Sigma to radius relation in logarithmic scale





The model



$$f(\sigma; A, r) = \frac{A\omega r\kappa}{\sqrt{1+\kappa^2\left(\frac{\sigma}{\omega r}-\frac{\omega r}{\sigma}\right)^2}}\left(\frac{\left(\frac{\sigma}{\omega r}\right)^2}{1+\left(\frac{\sigma}{\omega r}\right)^2}\right)^{\eta}$$

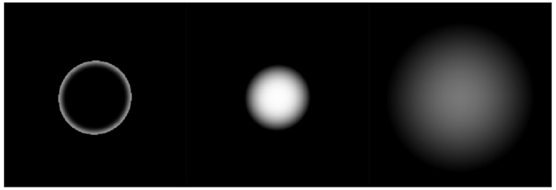
$\kappa = 17.289$, $\omega = 0.03411$ and $\eta = 432$

The algorithm

1. Select a point, preferably on the centerline of the vessel,
2. Compute vesselness at this point for multiple σ scales,
3. Fit the formula $f(\sigma; A, r)$ to the computed values,
4. Use the parameter r as an estimate of the radius

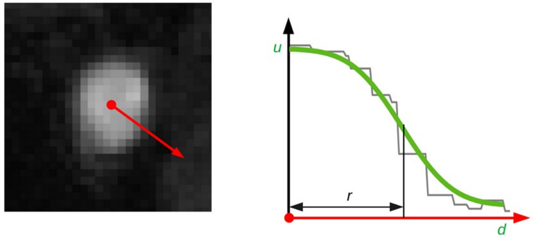
These steps are repeated for all points
along the centerlines.

Validation of the method

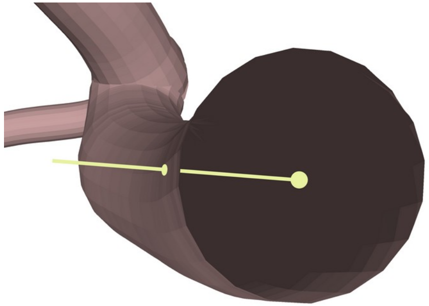


$$f(\sigma; A, r) = \frac{A\omega r\kappa}{\sqrt{1+\kappa^2\left(\frac{\sigma}{\omega r}-\frac{\omega r}{\sigma}\right)^2}}\left(\frac{\left(\frac{\sigma}{\omega r}\right)^2}{1+\left(\frac{\sigma}{\omega r}\right)^2}\right)^\eta$$

Radius estimation from multiscale vesselness (REMV)

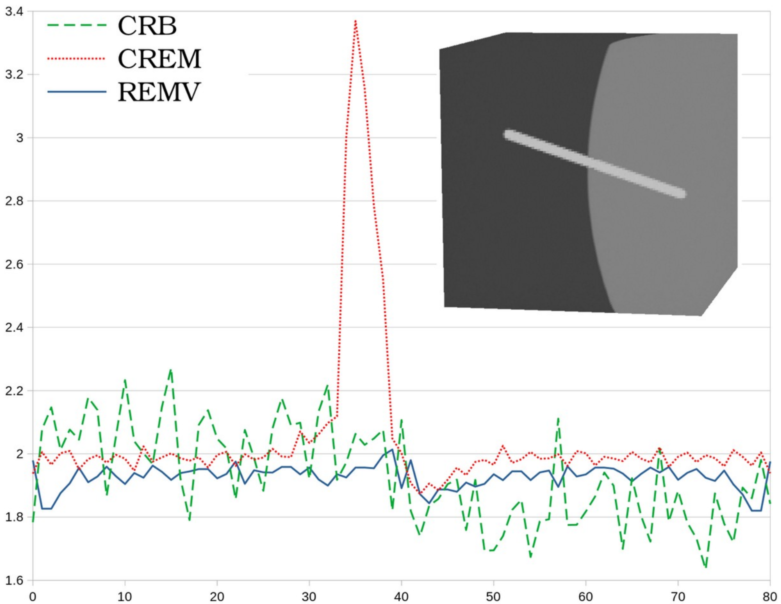


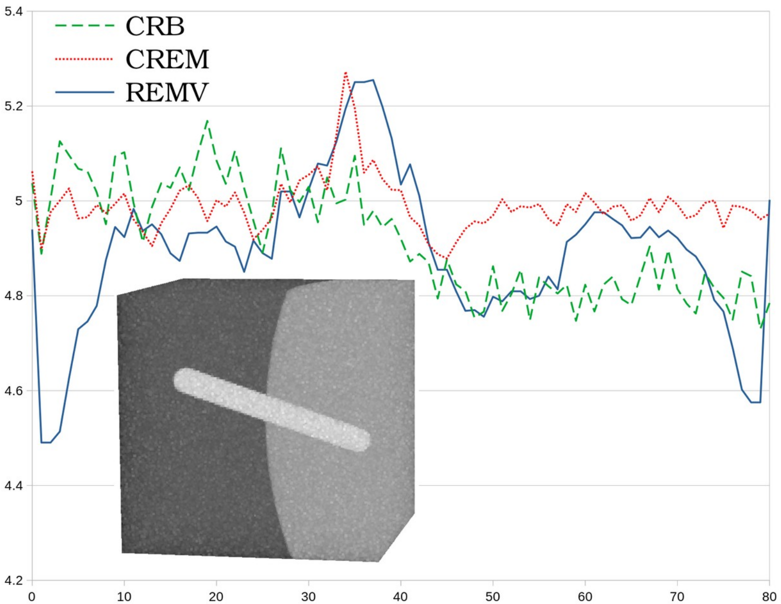
Cross-sectional ray-casting with erfc matching (CREM)



Cross-sectional ray-casting in binary image (CRB)

Artificial images of pipes





Artificial images of pipes

Radius	Noise	CREM	CRB	REMV
1	1	1.52	0.92	1.02
1	5	1.51	0.93	1.03
1	13	1.44	0.94	1.06
5	1	5.01	4.91	4.85
5	5	5.01	4.91	4.86
5	13	5.01	4.90	4.86
13	1	12.94	12.91	12.63
13	5	12.44	12.91	12.64
13	13	8.21	12.90	12.64
	Time:	02:06:41	00:00:39	00:18:36

Artificial images of pipes

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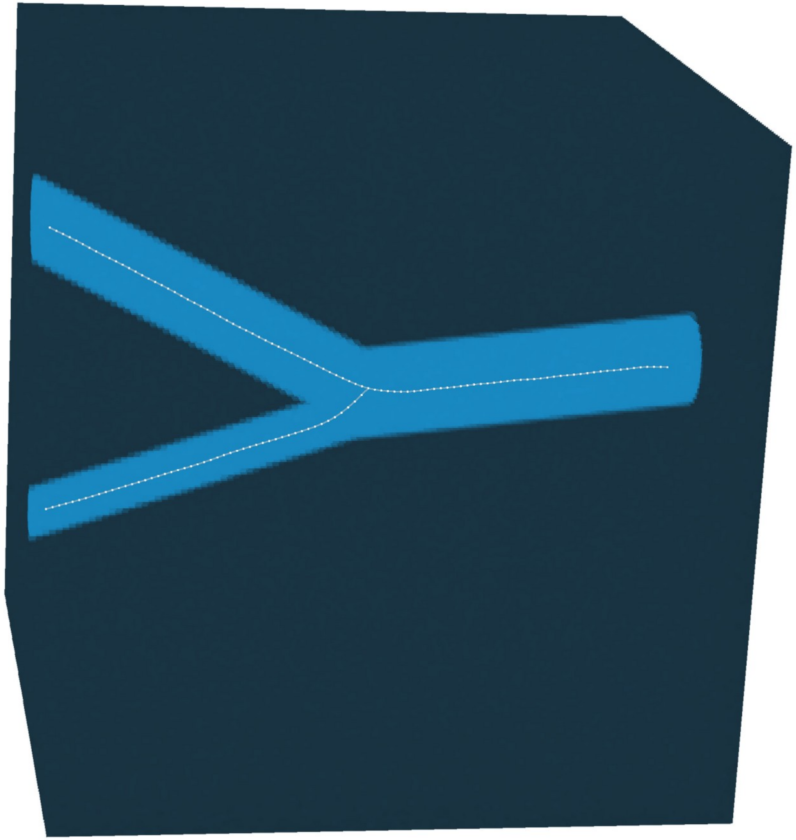
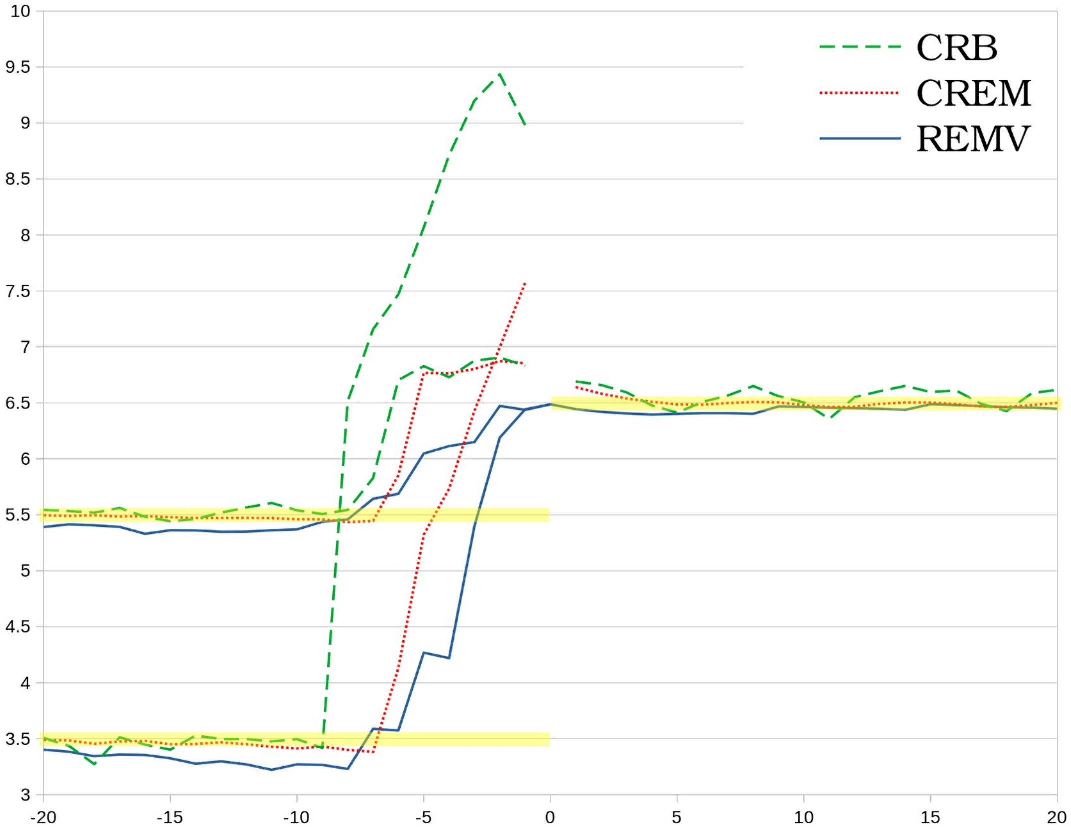
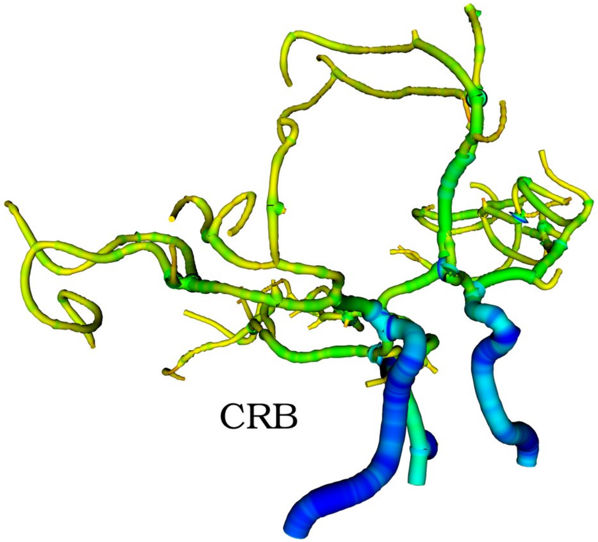
13	5	12.44	12.91	12.64
13	13	8.21	12.90	12.64
	Time:	02:06:41	00:00:39	00:18:36

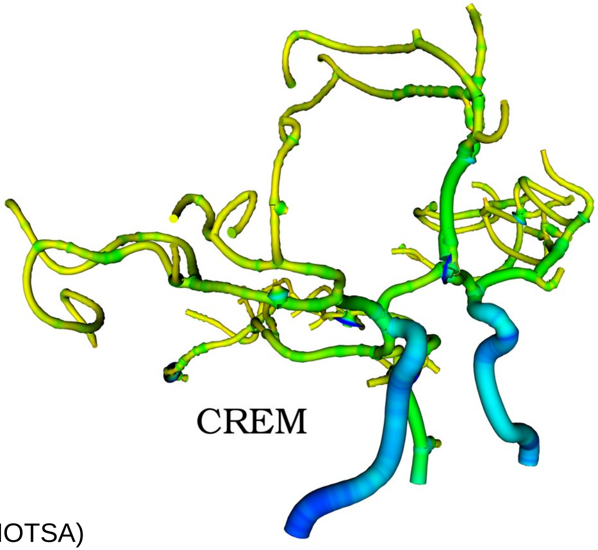
Bifurcation

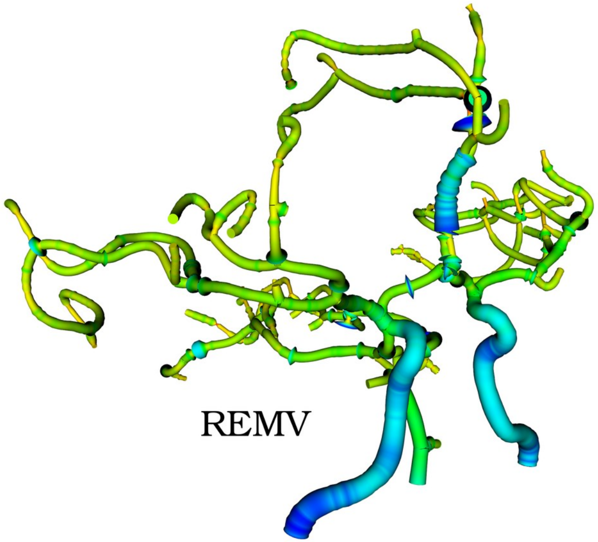




MRA







A 3D MRA of a head (TOF – MOTSA)
courtesy of Prof. Jürgen R. Reichenbach from University Hospital Jena, Germany

Summary

1. The equation relating the parameter σ to the radius r has been determined.
2. A new algorithm for estimating the radius has been developed.
3. The new algorithm has been validated and compared with reference methods.
4. The algorithms have been implemented and are available as open source.

http://www.eletel.p.lodz.pl/pms/SoftwareVesselKnife.html
https://gitlab.com/vesselknife/vesselknife/tree/master

Conclusions

The developed REMV algorithm:

1. Does not require computation of the vessel's cross-section.
2. Is computationally efficient.
3. Accurately estimates radii of relatively thin blood vessels.
4. Exhibits higher resistance to noise compared to the *erfc* fitting method.